

## Assignment 2

### (State Space and Pole Placement Controller)

#### **Problem 1 Pole Placement Controller**

**Goal:** To design a Pole Placement Controller (Polynomial Approach) and study the effect of different pole locations and disturbances.

**Problem 1.1:** Design a pole placement controller for the process given below. Find the polynomial  $C(z)$  and  $D(z)$ , and the gain factor  $K_r$  for the different values of the pole for the control system: (i)  $q=0.5$  (ii).  $q=0$  (iii).  $q=-0.2$

$$G(s) = \frac{\alpha}{1 + \beta s}$$

Let the sample interval be  $h = 2$  seconds.

How to select  $\alpha$  : Last three digits of your student id divided by 100

How to select  $\beta$  : Sum of last four digits of your student id

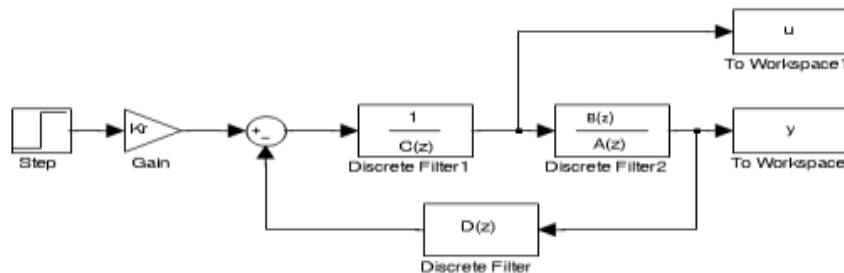
Example: Student ID= 202286135

$$\alpha = 135 \div 100 = 1.35$$

$$\beta = 6 + 1 + 3 + 5 = 15$$

**Problem 1.2:** Use the Matlab command `c2d` to find the corresponding discrete transfer function based on the continuous model in part a.

**Problem 1.3:** Create a control system block diagram using Simulink, as shown in the following figure.



Simulate the three cases given in part a. Plot the reference, output, and control signal for all cases. Notice the control signal is *constant* during the sample interval, and the output signal is only known at the *sample instances*. These circumstances must be considered when these signals are plotted. Comments all figure. Discus

also the influence of the location of the pole on the step response.

**Problem 1.4:** Let the reference be equal to zero. Add a step disturbance to the control signal. Perform the simulation using the same three controllers as designed in part a. Comment and discuss the influence of the disturbance.

## **Problem 2 Model Analysis**

**Goal:** Analyse the observability and controllability

**Background:** A steel ball roll on a track that can be inclined. The track is fixed at one end, but the other end can be raised or lowered. The movement of the end is done by an electrical motor, which rotation speed can be controlled by a control signal.

Practical, there is a nut fixed at the moving end of the track, and on the motor axis, there is a threaded rod. The motor is fixed, and the threaded axis is going through the nut. By reversing the rotation direction, the end of the track can be raised or lowered, see Figure 1.

The steel ball roll on two treads like a train on railway tracks. One of the threads is a common copper thread, while the other is a thread from a resistor. At the position of the ball, there will be a short circuit between the two threads. Hence, the position of the ball can be measured by a bridge circuit.

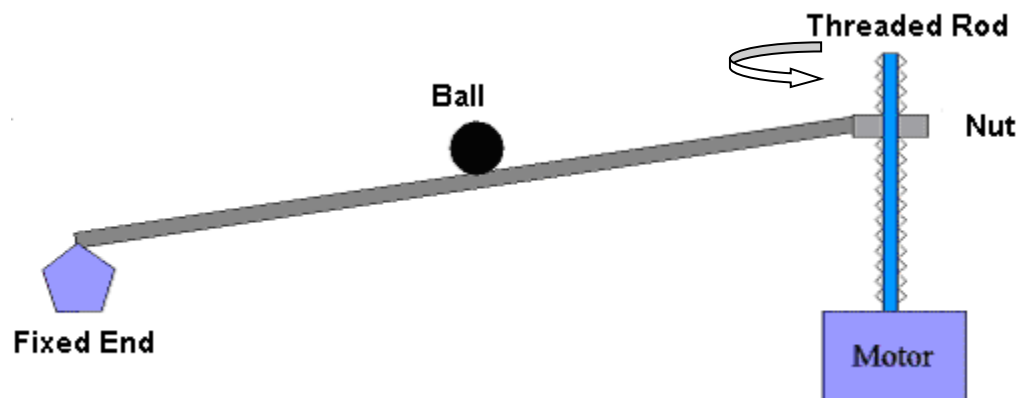


Figure 1 Ball system

### **Problem 2.1: State-space model**

A model of the ball system is constructed using Newton's second law:

$$m\ddot{x} = F$$

Where  $m$  is the mass of the ball and  $F$  is the force acting on the direction of the ball movement. See also Figure 2.

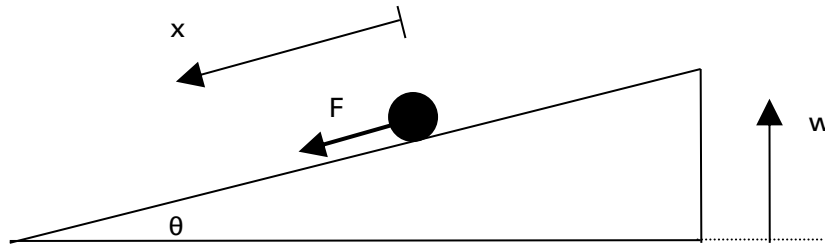


Figure 2 Schematic diagram of the ball system

The force acting on the system is given as:

$$F = mg \cdot \sin(\theta)$$

Approximatively, assume that the angle  $\theta$  is small:

$$\sin(\theta) = \frac{w}{L}$$

Where the length of the track is  $L = 1.0$  meter.

Hence, the position of the ball is described by:

$$\ddot{x} = \frac{g \cdot w}{L}$$

When the time constant of the motor is neglected, the relation between the control signal and the deviation of the moving end of the track can be described by:

$$\dot{w} = K \cdot u$$

Where  $K$  is a constant and is chosen as a value within:  $0.004 \leq K \leq 0.01$

$$x_1 = x$$

Introduce the states:  $x_2 = \dot{x}$  and the position of the ball as the output.

$$x_3 = w$$

Describe the system in the state space form:

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

### Problem 2.2: Discretisation

Transform the model into a discrete state-space form, where the sample interval is chosen as  $h = 1$  second. Use Matlab for the discretization.

$$x(k+1) = Fx(k) + Gu(k)$$

$$y(k) = Cx(k) + Du(k)$$

**Problem 2.3: Observability and controllability**

Analyze whether the system is possible to be observed and controlled. Use Matlab for computations.

*Motivate and discuss your conclusions!*

**Problem 2.4: Implementation using Simulink**

Implement the process using the *Discrete State-Space* block from Simulink. Introduce realistic restrictions on the control signal. Test the model by analyzing the step response.

**Problem 3 Observer Design**

**Goal:** Design and implement a model-based observer

**Problem 3.1 Design an observer**

Use the discrete model given in problem 2. Construct an observer which estimates all states.

*Discus and motivate your choice of observer poles.*

**Problem 3.2 Implement the observer using Simulink**

Implement the observer using suitable blocks from Simulink. Notice the possibility of using *Create subsystem*.

Test the observer by simulating the observer together with the process.

**Problem 4 State Space Controller Design**

**Goal:** Design and implement a model-based state base controller

**Problem 4.1 Design a state-space controller**

Use the discrete model given in problem 3. Construct a state-space controller which controls the system based on estimated states.

*Discus and motivate your choice of controller poles.*

**Problem 4.2 Implement the controller based on estimated states using Simulink**

Implement the controller based on estimated states using suitable blocks from Simulink. Notice the possibility of using *Create subsystem*.

**Problem 4.3 Simulation**

Simulate changes in the reference. Plot curves that describe interesting results.

Simulate also disturbances such as that the ball's position is moved to a location different from the reference point.

*Comment and discuss the results.*

**Problem 5 State Space Controller with Integration**

**Goal:** Design and implement a model-based state base controller with integration.

**Problem 5.1 Design a state-space controller with integration**

Augment the state space controller with integration action. Introduce also feedforward compensation which speeds up changes in the reference.

**Problem 5.2 Simulation**

Simulate changes in the reference.

Simulate different types of disturbances.

*Comment and discuss the results.*

**Problem 6 Own Problem**

**Goal:** Formulate and discuss your own problem

**Problem 6.1 Formulate your own problem**

Formulate and solve your own problem which is related to the application of these controllers in the energy area. The size of the problem should be about 20% of the total Assignment 2.

**Problem 6.2 Simulation**

Simulate your problem.

*Comment and discuss the results.*